Solving The Neutrosophic Shortest Path Problem in Triangular Fuzzy Numbers Using Metric Distance Ranking

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***Abstract*: This paper proposes a novel approach to finding the shortest path in a directed graph, where each arc length is represented by a Neutrosophic Triangular Fuzzy Number. A systematic procedure is designed to determine the optimal path. An illustrative example is provided to demonstrate the effectiveness of the proposed approach.**

***Keywords*: Shortest path, Neutrosophic Triangular Fuzzy Number (NRTFN), Chen and Cheng Metric distance, Fuzzy network.**

1. Introduction

The Shortest Path Problem (SPP) is a classical network optimization problem that typically involves real numbers as weights for the edges. However, fuzzy numbers can be used to model the problem when uncertainty is present, giving rise to the Fuzzy Shortest Path Problem (FSPP). Fuzzy set theory, proposed by Zadeh [12], is an effective tool for addressing problems involving uncertainty.

Determining the shortest distance and shortest path between two vertices is a fundamental problem in graph theory. Let G = (V, E) be a graph with V as the set of vertices and E as the set of edges. A path between two vertices is an alternating sequence of vertices and edges, starting and ending with vertices, with no repeated vertices or edges. The length of a path is the sum of the weights of the edges on the path.

There may be multiple paths between a pair of vertices. The SPP is a fundamental combinatorial optimization problem that appears in many applications as a sub-problem. The length of arcs in the network can represent travel time, cost, distance, or other variables.

Dubois and Prade [2] first introduced the concept of fuzzy SPP. Okada and Soper [4] developed an algorithm based on multiple labeling approaches, which generates multiple non-dominated paths. They introduced an order relation between fuzzy numbers using the minimum concept. Klein applied the extension principle to develop an algorithm that results in a dominated path on an acyclic network. R. Sophia Porchelvi and G. Sudha [6-12] developed an algorithm for critical path analysis in a project network.

This paper is organized as follows. In section 2, some preliminary concepts and definitions are given. The procedure for finding shortest path using N_RTFN is developed in section 3. An illustrative example is provided in section 4 to find the shortest path from a specified node to every node in a network having imprecise weights. The last section draws some concluding remarks.

2. Prerequisites

In this section, the basic concepts and definitions on neutrosophic sets, single valued neutrosophic sets, and Neutrosophic Triangular Numbers are reviewed from the literature.

A. Definition 2.1

Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of] -0, 1+ [. That is

$T_A(x) : X \to] -0, 1+ [$ $I_A(x) : X \to] -0, 1+ [$

and $F_A(x) : X \to] -0, 1+ [$.

There is no restriction on the sum $T_A(x)$, $I_A(x)$ of and $F_A(x)$, hence $0- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$

B. Definition 2

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$,

where $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy membership degree and the falsity membership degree of x to A, respectively.

For convenience, a single valued neutrosophic number is denoted by $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

C. Definition 3

Let $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3 \in \boldsymbol{R}$ such that $a_1 \leq a_2 \leq a_3$. Then a single valued Neutrosophic Triangular Number,

$\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special Neutrosophic set on the real line set $\boldsymbol{R}$, whose truth-membership, indeterminacy - membership, and falsity-membership functions are given as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\bar{a}}\left(\dfrac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\bar{a}}, x = a_2 \\ \alpha_{\bar{a}}\left(\dfrac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, otherwise \end{cases} \quad \text{------(1)}$$

$$I_{\tilde{a}}(x) = \begin{cases} \left(\dfrac{a_2 - x + \theta_{\bar{a}}(x-a_1)_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \theta_{\bar{a}}, x = a_2 \\ \left(\dfrac{x - a_2 + \theta_{\bar{a}}(a_3-x)}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 1, otherwise \end{cases} \quad \text{--- (2)}$$

$$F_{\tilde{a}}(x) = \begin{cases} \left(\dfrac{a_2 - x + \beta_{\bar{a}}(x-a_1)_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \beta_{\bar{a}}, x = a_2 \\ \left(\dfrac{x - a_2 + \beta_{\bar{a}}(a_3-x)}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 1, otherwise \end{cases} \quad \text{----- (3)}$$

Where $\alpha_{\bar{a}}$, $\theta_{\bar{a}}$ and $\beta_{\bar{a}}$ denote the maximum truth membership degree, minimum indeterminacy membership degree and minimum falsity membership degree respectively.

D. Definition 4

The Graded mean Index approach for Neutrosophic Triangular Fuzzy Number is

$$P^G(R) = \frac{m_1 + 2m_2 + m_3}{4}$$

E. Definition 5

Chen and Cheng [1] proposed a metric distance method to rank fuzzy numbers.

$$q_A{}^L(x) = (\lambda - \gamma) + \gamma x$$

$$q_A{}^R(x) = (\lambda + \gamma) - \gamma x$$

Where

$$\lambda = \frac{h-l}{2}; \gamma = \frac{l+2m+h}{4}$$

And

$$R(P) = \left[\int_0^1 (\gamma+\lambda)^2 dx + \int_0^1 (\gamma+\lambda)^2 dx\right]^{1/2}$$

3. Proposed Algorithm

Algorithm for Neutrosophic Triangular Fuzzy Shortest Path Problem based on Chen and Cheng [1].

A. Metric Distance Ranking

Step 1: Construct a network G= (V, E) where V is the set of vertices E is the set of edges. Here G is an acyclic digraph and the arc length takes the Neutrosophic Triangular Fuzzy Numbers.

Step2: Form the possible paths Pi from source vertex to the destination Vertex and compute the corresponding path lengths Li, using 2.1.

Step3: Calculate the Neutrosophic Triangular Fuzzy path using2.2.

Step4: The path having the minimum rank is identified as the shortest path.

4. Illustrative Example

Consider a network with Neutrosophic Triangular fuzzy arc lengths as shown below.

The arc lengths are assumed to be

$$A(1-2) = [(1,2,3),(3,4,5),(4,5,6)]$$
$$B(1-3) = [(0,1,6),(0,2,8),(2,3,8)]$$
$$C(2-5) = [(0,2,4),(2,3,4),(4,6,8)]$$
$$D(3-4) = [(0,4,8),(3,4,9),(2,8,10)]$$
$$E(4-5) = [(1,2,7),(0,3,10),(2,8,10)]$$
$$F(4-6) = [(0,2,4),(1,2,7),(2,4,6)]$$
$$G(5-6) = [(0,1,2),(0,2,4),(3,4,5)]$$

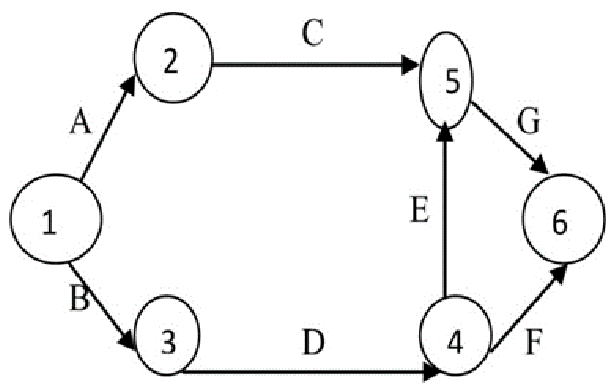

Fig. 1. Connected path graph

The possible paths and its corresponding path lengths are as follows

Table 1
Lengths (L_i) using graded mean

Paths (P_i)	Connections	Graded Mean Index
P_1	1−2−5−6	(5, 9, 15)
P_2	1−3−4−5−6	(10, 14,22)
P_3	1−3−4−6	(8, 11, 15)

To obtain the shortest Path using Chen and Cheng [1] Metric distance

$$P_1 = (5,9,15)$$

$$\gamma = \frac{H-L}{2} = \frac{15-5}{2} = 5$$

$$\lambda = \frac{L+2M+H}{4} = \frac{5+2(9)+15}{4} = 9.5$$

$$q_A^{\ L}(x) = (\lambda - \gamma) + \gamma x = 4.5 + 5x$$

$$q_A^{\ R}(x) = (\lambda + \gamma) - \gamma x = 14.5 - 5x$$

Hence R(P1) = 14.25.

Similarly, R(P2) = 21.7 and R(P3) = 16.23.

Since R(P1) < R(P3) < R(P2). From the above analysis, it is clear that path P1 has the least value. Therefore, the shortest path from source node to destination node is 1−2−5−6.

5. Conclusion

In this paper, an algorithm is developed for solving SPP on a network with Neutrosophic Triangular fuzzy arc lengths. The shortest path is identified using Chen and Cheng's [1] ranking function, which enables the decision-maker to select the best path from among various alternatives based on the ranking list.

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