The Shortest Path Problem in Neutrosophic Triangular Fuzzy Environment

G Sudha[1]*, P Jamunadevi[1], R Sophia Porchelvi[2], R Vanitha[2], R Seema[3], C Jayabharathi[3]

[1]*Assistant Professor, PG & Research Department of Mathematics, A.D.M. College for Women (Autonomous), Affiliated to Bharathidasan University, Nagapattinam. Tamil Nadu, India.*

[2]*Associate Professor, PG & Research Department of Mathematics, A.D.M. College for Women (Autonomous), Affiliated to Bharathidasan University, Nagapattinam. Tamil Nadu, India.*

[3]*Research scholar, PG & Research Department of Mathematics, A.D.M. College for Women (Autonomous), Affiliated to Bharathidasan University, Nagapattinam. Tamil Nadu, India.*

Corresponding Author: sudha1994ganesan@gmail.com

***Abstract*: This paper introduces a new type of fuzzy shortest path network problem using triangular fuzzy number. Using the graded mean integration form of the generalized fuzzy number for each node, the smallest edge is identified based on fuzzy distance, thereby determining the optimal shortest path for the problem.**

***Keywords*: Fuzzy Number, Graded Mean Integration Representation, Neutrosophic Fuzzy Distance, Shortest path problem.**

1. Introduction

The shortest path problem was one of the first network problems studied in the context of operations research. In some applications, the numbers associated with the edges of networks may represent characteristics other than lengths, and we may seek the optimum paths, where the optimum can be defined by various criteria. The shortest-path problem is the most common problem within the broader class of optimum path problems. Consider the edge weight of the network as uncertain, meaning it is either imprecise or unknown.

In 1965, Zadeh [22] introduced the concept of fuzzy set theory to address these problems. In 1978, Dubois and Prade [3] defined a fuzzy number as a fuzzy subset of the real line. In 1997, Heilpern [5] proposed three definitions of the distance between two fuzzy numbers, including the mean distance method, which is generated by the expected values of fuzzy numbers; the distance method, which combines a Minkowski distance and the h-levels of closed intervals of fuzzy numbers; and the geometrical distance method, which is based on the geometrical operation of trapezoidal fuzzy numbers. All of these methods use real numbers to calculate the distance. Here, the fuzzy distance between two trapezoidal fuzzy numbers is measured using graded mean integration representation and fuzzy numbers. R. Sophia Porchelvi and G. Sudha [7-22] developed a shortest path problem using various ranking fuzzy techniques.

2. Prerequisites

In this section, the basic concepts and definitions on neutrosophic sets, single valued neutrosophic sets, and Neutrosophic Triangular Numbers are reviewed from the literature.

Definition 2.1

Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $\mathrm{TA}(x)$, an indeterminacy-membership function $\mathrm{IA}(x)$ and a falsity-membership function $\mathrm{FA}(x)$. $\mathrm{TA}(x)$, $\mathrm{IA}(x)$ and $\mathrm{FA}(x)$ are real standard or real nonstandard subsets of] -0, 1+ [. That is

$\mathrm{TA}(x) : X \rightarrow]$ -0, 1+ [$\mathrm{IA}(x) : X \rightarrow]$ -0, 1+ [

and $F\mathrm{A}\ (x) : X \rightarrow]$ -0, 1 + [.

There is no restriction on the sum $T\mathrm{A}(x)$, $I\mathrm{A}(x)$ of and $F\mathrm{A}(x)$, hence $0- \leq \sup T\mathrm{A}(x) + \sup I\mathrm{A}(x) + \sup F\mathrm{A}(x) \leq 3+$

Definition 2.2

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$A = \{\langle x, T\mathrm{A}(x), I\mathrm{A}(x), F\mathrm{A}(x)\rangle : x \in X\}$,

where $T\mathrm{A}(x)$: $X \rightarrow [0,1]$, $I\mathrm{A}(x)$: $X \rightarrow [0,1]$ and $F\mathrm{A}(x)$: $X \rightarrow [0,1]$ with $0 \leq T\mathrm{A}(x) + I\mathrm{A}(x) + F\mathrm{A}(x) \leq 3$ for all $x \in X$. The intervals $T\mathrm{A}(x)$, $I\mathrm{A}(x)$ and $F\mathrm{A}(x)$ denote the truth-membership degree, the indeterminacy membership degree and the falsity membership degree of x to A, respectively.

For convenience, a single valued neutrosophic number is denoted by $A = (a, \mathrm{b}, \mathrm{c})$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

Definition 2.3

Let $\mathrm{a}\tilde{a}, \mathrm{q}\tilde{a}, \mathrm{b}\tilde{a} \in [0,1]$ and $a1, a2, a3 \in \boldsymbol{R}$ such that $a1 \leq a2 \leq a3$. Then a single valued Neutrosophic Triangular Number,

$\tilde{a} = \langle (a1\ , a2\ , a3); \mathrm{a}\tilde{a}\ , \mathrm{q}\tilde{a}\ , \mathrm{b}\tilde{a}\ \rangle$ is a special Neutrosophic set on the real line set $\boldsymbol{R}$, whose truth-membership, indeterminacy - membership, and falsity-membership functions are given as follows:

$$\begin{cases} \alpha_{\bar{a}}\left(\dfrac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \alpha_{\bar{a}}, x = a_2 \\ \alpha_{\bar{a}}\left(\dfrac{a_3-x}{a_3-a_2}\right), a_2 < x \le a_3 \\ 0, otherwise \end{cases} \quad \text{------(1)}$$

$$I_{\tilde{a}}(x) = \begin{cases} \left(\dfrac{a_2-x+\theta_{\bar{a}}(x-a_1)_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \theta_{\bar{a}}, x = a_2 \\ \left(\dfrac{x-a_2+\theta_{\bar{a}}(a_3-x)}{a_3-a_2}\right), a_2 < x \le a_3 \\ 1, otherwise \end{cases} \quad \text{--- (2)}$$

$$F_{\tilde{a}}(x) = \begin{cases} \left(\dfrac{a_2-x+\beta_{\bar{a}}(x-a_1)_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \beta_{\bar{a}}, x = a_2 \\ \left(\dfrac{x-a_2+\beta_{\bar{a}}(a_3-x)}{a_3-a_2}\right), a_2 < x \le a_3 \\ 1, otherwise \end{cases} \quad \text{----- (3)}$$

Where $\alpha_{\bar{a}}$, $\theta_{\bar{a}}$ and $\beta_{\bar{a}}$ denote the maximum truth membership degree, minimum indeterminacy membership degree and minimum falsity membership degree respectively.

Definition 2.4

The Graded mean Index approach for Neutrosophic Triangular Fuzzy Number is

$$P^G(R) = \frac{m_1 + 2m_2 + m_3}{4}$$

Definition 2.5

Let X= (a_1, a_2, a_3) and H(Y) be the reduced fuzzy number, and their Graded mean integration representation are H(X) , H(Y) respectively.

Assume

Si = (ci –H(X)+ di –H(Y))/2, i =1,2,3; for each adjacent node

Ci = | H(X) – H(Y) | + G_i, i =1,2,3;

then the fuzzy distance of X, Y is $V_1 = (v_1, v_2, v_3)$.

3. Proposed Algorithm

Step-1: Assume R = [(0,0,0), (0,0,0), (0,0,0)] then compute fuzzy distance.

Step -2: Calculate the Graded mean value for each adjacent vertex from the current vertex. H(X) = [a+ 4b + c] / 6

Step-3: Evaluate the neutrosophic fuzzy distance between two edges using

$G_i = [\, c_i - H(X) + d_i - H(Y)] / 2$, i = 1,2,3 and

$V_i = \int H(X) - H(Y) \int + G_i$, i = 1,2,3 and

then the neutrosophic fuzzy distance of X, Y is V = (v_1, v_2, v_3) for each adjacent node.

Step -4: Compare Neutrosophic fuzzy distance among all adjacent nodes

Step -5: Select the smallest Neutrosophic fuzzy distance edge among them and continue until destination node is reached.

Step- 6: Neutrosophic fuzzy shortest path is obtained for the given network.

4. Numerical Example

The problem is to find the shortest path between source node (ie. Node 1) and destination node (ie. Node 6) on the network consists of 6 vertices and 7 edges, the arc lengths of the network shown in the following table are all NRTFN and given by

Table.1.
Convert Neutrosophic Triangular Fuzzy Number to Triangular Fuzzy number

Edges	Neutrosophic Triangular Fuzzy Number	Triangular fuzzy using Graded Mean Integration (2.4)
1-2	[(1,2,3), (3,4,5), (4,5,6)]	(2,4,5)
1-3	[(0,1,6), (0,2,8), (2,3,8)]	(2,3,4)
1-4	[(0,1,2), (1,2,3), (2,3,4)]	(1,2,3)
2-5	[(0,2,4), (2,3,4), (4,6,8)]	(2,3,6)
3-6	[(1,2,7), (0,3,10), (2,8,10)]	(3,4,7)
4-6	[(0,2,4), (1,2,7), (2,4,6)]	(2,3,4)
5-6	[(0,1,2), (0,2,4), (3,4,5)]	(1,2,4)

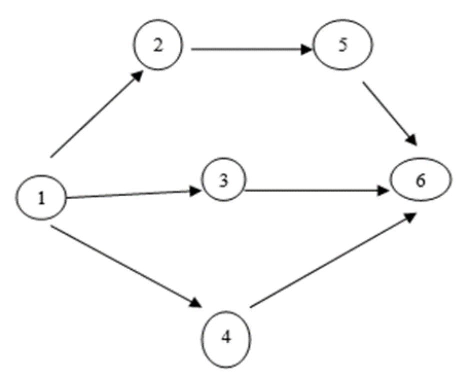

Fig.1. Neutrosophic Fuzzy Network

Now start with node 1, the distance

R = [(0,0,0), (0,0,0), (0,0,0)]. From node 1, the adjacent nodes are 2,3 and 4.

H (1,1) = [(0,0,0), (0,0,0), (0,0,0)] = $\frac{0}{6}$ = 0

$H(1,2) = \frac{2+4(4)+5}{6} = 3.83$

$G_1 = \frac{0-0+2-3.83}{2} = -0.92;$

$G_2 = \frac{0-0+4-3.83}{2} = 0.085;$

$G_3 = \frac{0-0+5-3.83}{2} = 0.585$

$v_1 = \int 0 - 3.83 \int +(-0.915) = 2.92;$

$v_2 = \int 0 - 3.83 \int +(0.085) = 3.92;$

$v_3 = \int 0 - 3.83 \int +(0.585) = 3.25$

Hence $V_1 = (2.915, 3.92, 4.42)$

Similarly,

$V_2 = (3.5,3,2.5)$ and $V_3 = (2.5, 2, 1.5)$

We compare (1,2), (1,3) and (1,4) Neutrosophic fuzzy distance edges, we get (1,4) is the least among them to order relation. From node 4, the only one node is 6. Then we get the shortest path 1-4-5. It can be easily seen that the procedure is very simple and needs light computational load.

5. Conclusion

This paper defines a solution for a shortest path problem with N_RTFN. The main issue dealt with was developing a fuzzy distance for N_RTFN and with minimum number of process steps. The algorithm can return a single path as the solution and can be implemented using fuzzy numbers graded mean integration selected by the decision maker This algorithm is tested on a variety of networks. For several network kinds, it offers superior output. It offers superior results for many network categories.

References

[1] Bortolan G. and Degani. R, 1985, “A review of some methods for ranking fuzzy subsets”, Fuzzy sets and Systems, Vol.15, pp.1-19.
[2] S. H. Chen and C. H. Hsieh, 1998, “Graded mean integration representation of generalized fuzzy number”, Proc. Of Conference of Taiwan Fuzzy System Association, Taiwan.
[3] Dubois. D and H. Prade 1980, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York.
[4] S. Heilpern, 1997, “Representation and application of fuzzy numbers”, Fuzzy sets and Systems, Vol. 91, No.2, pp.259- 268.
[5] Shan Huo Chen and Chih Hsun Hsieh, 1999, “Graded Mean Integration Representation of Generalized Fuzzy Number”, Journal of the Chinese Fuzzy System Association, Vol. 5, No.2, pp. 1-7.
[6] Shan Huo Chen and Chih Hsun Hsieh, 2000, “Representation, Ranking, Distance, and Similarity of L-R type fuzzy number and Application”, Australia Journal of Intelligent Information Processing Systems, Vol. 6, No. 4, pp.217-229.
[7] R. Sophia Porchelvi, G. Sudha, A new approach for finding Minimum path in a network using triangular intuitionistic fuzzy number, International Journal of Current Research, 6(8), 8103- 8109, 2014.
[8] G. Sudha, R. Sophia Porchelvi, K. Gnanaselvi, International Journal of Fuzzy Mathematical Archive, 14(1), 1- 8, 2017.
[9] [9] R. Sophia Porchelvi, G. Sudha, A Modified algorithm for solving shortest path problem with intuitionistic fuzzy arc length, International Journal of Engineering Research, 4(10), 2013.
[10] R. Sophia Porchelvi, G. Sudha, Crtical path analysis in a project network Using Ranking method in intuitionistic fuzzy Environment, International Journal of Advances Research, 3, 14 – 20, 2015.
[11] R. Sophia Porchelvi, G. Sudha, An intuitionistic fuzzy Critical path Problem using Ranking method, International Journal of Current Research, 8(2), 44254 – 44257, 2016.
[12] R. Sophia Porchelvi, G. Sudha, Intuitionistic fuzzy critical path in a network, Proceedings of International Mathematical methods and Computations, 2014.
[13] R. Sophia Porchelvi, G. Sudha, Computation of shortest path in a fuzzy network using triangular intuitionistic fuzzy number, International Journal of Scientific and Engineering Research, 5(12), 1254- 1257, 2014.
[14] R. Sophia Porchelvi, G. Sudha, Modified approach on shortest path in Intuitionistic fuzzy Environment, International Journal of Applied Research, 4(9), 341—342.2014
[15] R. Sophia Porchelvi, G. Sudha, Transportation problem in evaluating efficiency of transport Hubs with Special Reference to Tamilnadu, International Journal of Engineering Research and Technology, vol. 14, 2278-0181, 2025
[16] G. Sudha, P. Jamuna Devi, R. Sophia Porchelvi, R. Vanitha, R. Seema, Solving the Neutrosophic Shortest Path Problem in Triangular Fuzzy numbers using Metric Distance Ranking, International Journal of Progressive Research in Science and Engineering,6(4), 13 – 15, 2025.
[17] G. Sudha, Rahini, K. Arumugam, Critical Path problem on a fuzzy network with Triangular Intuitionistic fuzzy Number, International Journal of Trend in Research and Development,4(1), 469 – 471, 2017.
[18] G. Sudha, R. Sophia Porchelvi, S. Vishnupriya, A novel approach for finding shortest path in Intuitionistic fuzzy network, International Journal of Fuzzy Mathematical Archive, 14(1), 139-146, 2017.
[19] R. Sophia Porchelvi, G. Sudha, Shortest path problem in a network using Intuitionistic fuzzy number – a case study about Kanyakumari Roadways network, Elixir, Appl. Math., Vol.82, 32655 – 32657, 2015.
[20] R. Sophia Porchelvi, G. Sudha, An approach to find the Optimal paths using trapezoidal Intuitionistic fuzzy number, Bonfring, NCRAMAA2013,978-93-82338-70-3, 356-360, 2013.
[21] R. Sophia Porchelvi, G. Sudha, Critical path problem under Intuitionistic fuzzy Environment, Int. Journal of Applied Engineering Research, 10(51), 2015.
[22] A. Zadeh, 1965, “Fuzzy set”, Information and Control, Vol.8, pp. 338-353.