Numerical Solution of the Ninth Order Linear Differential Equation Using Ninth Degree B-Spline Collocation Method

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***Abstract*: In this paper, Collocation method using recursive form of Ninth degree B-spline functions as basis is developed and employed to find the numerical solution for ninth order boundary value problems. Numerical examples are considered to test the performance, stability and accuracy of the present developed method.**

***Keywords*: B-Spline, Collocation, Recursive, Linear differential equation.**

1. Introduction

This paper is concerned with the numerical solution of ninth order linear boundary value problem by using ninth degree B-spline collocation solution.

The ninth order linear differential equation with boundary conditions is given as

$$P0(x)\frac{d^9U}{dx^9}+P_1(x)\frac{d^8U}{dx^8}+P_2(x)\frac{d^7U}{dx^7}+P_3(x)\frac{d^6U}{dx^6}+P_4(x)\frac{d^5U}{dx^5}+P_5(x)\frac{d^4U}{dx^4}+P_6(x)\frac{d^3U}{dx^3}+P_7(x)\frac{d^2U}{dx^2}+P_8(x)\frac{dU}{dx}+P_9(x)U=Q(x)$$

$$x\in(a,b) \qquad (1)$$

with the boundary conditions

$$U(a)=d1, U(b)=d2U'(a)=d3, U'(b)=d4, U''(a)=d5, U''(b)=d6, U'''(a)=d7, U'''(b)=d8, U^{iv}(b)=d9 \qquad (2)$$

2. Introduction

The document is a template for Microsoft *Word* versions 6.0 or later. where $\boldsymbol{a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7,d8,d9}$ are constant. $P_0(x)$, $P_1(x), P_2(x)$, $\boldsymbol{P_3(x), P_4(x)}$, $\boldsymbol{P_5(x), P_6(x)}$, $\boldsymbol{P_7(x), P_8(x), P_9(x), Q(x)}$, are function of $\boldsymbol{x}$.

3. Description of Method

The solution domain a ≤ x ≤ b is partitioned into a mesh of uniform length h = $x_{j+1} - x_j$, where j=0,1,2,…, N-1,N . Such that a= $x_0 < x_1 < x_2 \ldots\ldots < x_{n-1} < x_n$=b.

In the ninth degree B-spline collocation method the approximate solution is written as the linear combination of ninth degree B-spline basis functions for the approximation space under consideration. The proposed numerical solution for solving Eq. (1) using the collocation method with ninth degree B-spline is to find an approximation solution $U^h(x)$ to the exact solution $U(x)$ in the form:

$$U^h(x)=\sum_{i=-8}^{n+8}C_iN_{i,p}(x) \qquad (3)$$

where C_i's are constants to be determined from the boundary conditions and collocation from the differential equation.

A zero degree and other than zero-degree B-spline basis functions [3, 4] are defined at x_i recursively over the knot vector space$X=\{x_1,x_2,x_3\ldots\ldots\ldots x_{n-1},x_n\}$ as

$i) if p=0$

$N_{i,p}(x)=1 \quad if \quad x\in(x_i,x_{i+i})$

$N_{i,p}(x)=0 \quad if \quad x\notin(x_i,x_{i+i})$

$ii) if p\geq 1$

$$N_{i,p}(x)=\frac{x-x_i}{x_{i+p}-x_i}N_{i,p-1}(x)+\frac{x_{i+p+1}-x}{x_{i+p+1}-x_{i+1}}N_{i+1,p-1}(x)$$

………(4)

where p is the degree of the B-spline basis function and $x$is the parameter belongs to$X$.When evaluating these functions, ratios of the form 0/0 are defined as zero.

Derivatives of B-splines

If p=9, we have

$$\boldsymbol{N'_{i,p}(x)=\frac{x-x_i}{x_{i+p}-x_i}N'_{i,p-1}(x)+\frac{N_{i,p-1}(x)}{x_{i+p}-x_i}+\frac{x_{i+p+1}-x}{x_{i+p+1}-x_{i+1}}N'_{i+1,p-1}(x)-\frac{N_{i+1,p-1}(x)}{x_{i+p+1}-x_{i+1}}}$$

$$N^{ix}{}_{i,p}(x)=9\frac{N^{viii}{}_{i,p-1}(x)}{x_{i+p}-x_i}-9\frac{N^{viii}{}_{i+1,p-1}(x)}{x_{i+p+1}-x_{i+1}}\ldots(5)$$

$$(U^h)^{ix}(x) = \sum_{i=-9}^{n+9} C_i N^{ix}{}_{i,p}(x)\ldots\ldots\ldots(6)$$

The $x_i's$ are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns $C_i's$ in (3).Nine extra knots are taken into consideration besides the domain of problem to maintain the partition of unity when evaluating the ninth degree B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (3) to (6) in equation (1) for U (x) and derivatives of U (x). Then system of (n+1) linear equations are obtained in (n+9) constants. Applying the boundary conditions to equation (2), eight more equations are generated in constants. Finally, we have (n+10) equations in (n+10) constants.

Ninth Degree B-spline collocation Method for Numerical Solution of the ninth order Linear Differential Equation

Solving the system of equations for constants and substituting these constants in equation (3) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points.

This is implemented using the Matlab programming.

4. Numerical Scheme

Example1

A ninth order differential equation with boundary conditions [5] is considered to test the performance of the proposed method.

$$\frac{d^9y}{dx^9} - y = -9e^x$$

with the boundary conditions $y(0) = 1$ $y(1) = 0$ $y'(0) = 0$ $y'(1) = -e$ $y''(0) = -1$ $y''(1) = -2e$ $y'''(0) = -2$ $y'''(1) = -3e$ $y^{iv}(1) = -3$

The exact solution for the example is $y = x(1-x)e^x$

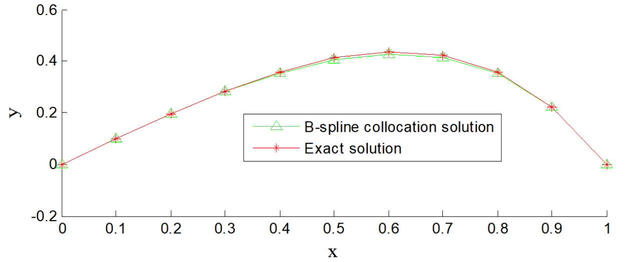

Fig. 1. Comparison of ninth degree B-spline collection solution with exact solution for 11 collocation points

The obtained numerical solution is compared with the exact solution which is shown graphically in figure1.Absolute relative errors are evaluated at different node points and presented in table1.

Table 1
Absolute relative errors at nodes

Nodes	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
Absolute relative errors	0	0.00	.0010	.0043	0.0097	0.0156	0.0194	0.0186	0.0130	0.0048

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Example 2

$$\frac{d^9y}{dx^9} + \frac{d^7y}{dx^7} + x\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + \sin x\frac{dy}{dx} + y = 5x\sin x - \cos x + x^2\cos x - x\sin^2 x + \sin x\cos x + x\cos x$$

with the boundary conditions $y(0) = 0$ $y(1) = \cos 1$ $y'(0) = 1$ $y'(1) = \cos 1 - \sin 1$ $y''(0) = 0$ $y''(1) = -2\sin 1 - \cos 1$ $y'''(0) = -3$ $y'''(1) = -3\cos 1 + \sin 1$ $y^{iv}(1) = 0$

The exact solution for example 2 is given as $y = x\cos x$

Clearly from table1 we have seen that the B-spline collocation solution values are meeting with the exact values accurately. Absolute relative errors are presented graphically in figure2.

Table 2
Comparison of B-spline collocation solution with exact solution

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
Ninth degree B-spline collocation solution	0.000	.0995	.1960	.2866	.3684	.4387	.4952	.5354	.5574	.5594
Exact solution	0.000	.0995	.1960	.2866	.3684	.4388	.4952	.5354	.5574	.5594

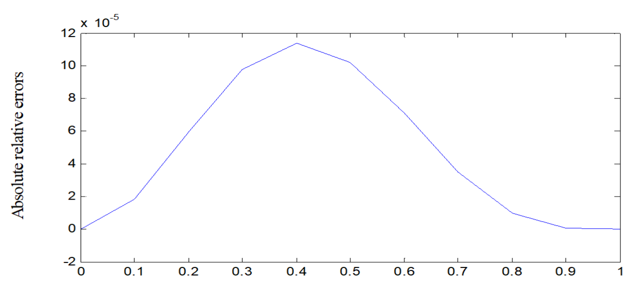

Fig. 2. Ninth Degree B-spline collocation Method for Numerical Solution of the ninth order Linear Differential Equation

5. Conclusions

In this article, developed collocation method by using the ninth degree B-spline as basis function in collocation method is applied to ninth order linear differential equations with boundary condition problems. It is observed that obtained values are very close to the exact values and further absolute relative errors are very less at the nodes. This

shows that the proposed method is effective and useful to find the numerical solutions for ninth order linear differential equation with boundary value problems.

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